We describe a device where the nonlocal spin-spin interaction between quantum dots (QDs) can be turned on and off with a small magnetic field. The setup consists of two QDs at the edge of two two-dimensional electron gases (2DEGs). The QDs' spins are coupled through a RKKY-like interaction mediated by the electrons in the 2DEGs. A magnetic field \( B_z \) perpendicular to the plane of the 2DEG is used as a tuning parameter. When the cyclotron radius is commensurate with the interdot distance, the spin-spin interaction is amplified by a few orders of magnitude. The sign of the interaction is controlled by finely tuning \( B_z \). Our setup allows for several dots to be coupled in a linear arrangement and it is not restricted to nearest-neighbor interaction.

Quantum information processing requires control and operation of interacting quantum mechanical objects [1]. One possibility is to produce systems with localized spins in atomic impurities, molecules or quantum dots (QDs) and manipulate the spin-spin interaction by engineering the electronic wave functions of the surrounding material [2–4]. This would allow for the nonlocal control of spins opening new possibilities for the fast developing field of spintronics [5]. An important step in this direction was reported very recently by Craig et al. [6], who coupled two QDs through a confined two-dimensional electron gas (2DEG) (a larger QD) and controlled the magnitude of the interaction by closing or opening up the QDs. Besides its relevance for spintronics, this experiment also opens up the possibility to study the interplay between two competing many-body effects: the Kondo effect and the RKKY-interaction [7–9].

In this Letter, we propose a different device where the magnitude (and sign) of the spin-spin interaction between two QDs can be tuned by a external field. Our setup consists of two QDs at the edge of two semi-infinite 2DEGs. When each dot is gated to have an odd number of electrons, and therefore a total spin \( 1/2 \), they interact through the polarization of the 2DEG as in the usual case described by the RKKY-interaction [10]. We show that this interaction can be controlled by applying a small magnetic field perpendicular to the plane of the 2DEG. The effect relies on the existence of edge states. These are responsible for the transverse focusing of electrons injected from a point contact [11–13] or QD [14]. The control mechanism is based on the possibility of focusing the electrons that interact with one QD onto the other. When the cyclotron radius is commensurate with the interdot distance, the spin-spin interaction is largely amplified and may increase a few orders of magnitude. The mechanism can be extended to many spins and to new geometries that allow for the independent control of different pair interactions.

Consider the geometry shown in Fig. 1. It consists of a 2DEG with a small magnetic field \( B_z \) perpendicular to the plane containing the carriers. The 2DEG is divided in two half planes by a set of gate contacts which are also used to define the QDs as schematically shown in the figure. The Hamiltonian of the system reads

\[
\hat{H} = \hat{H}_{\text{QD}} + \hat{H}_{\text{2DEG}} + \hat{H}_T,
\]

where the first term is the Hamiltonian of the QDs

\[
\hat{H}_{\text{QD}} = \sum_{\alpha,\sigma} E_{\alpha\sigma} d_{\alpha\sigma}^\dagger d_{\alpha\sigma} + U_\alpha d_{\alpha\uparrow}^\dagger d_{\alpha\downarrow}^\dagger d_{\alpha\downarrow} d_{\alpha\uparrow} + \text{H.c.}
\]

Here \( \alpha = 1 \) and \( 2 \) refer to the left and right QD, respectively; \( d_{\alpha\sigma}^\dagger \) creates an electron with spin \( \sigma \) and energy \( E_{\alpha\sigma} \) in the QD labeled by \( \alpha \), and \( U_\alpha \) is the Coulomb energy defined by the capacitances of the system [15]. The single particle energies \( E_{\alpha\sigma} \) (measured from the Fermi energy, \( E_F \)) can be varied with a gate voltage \( V_\alpha \) as shown in Fig. 1. The second term in Hamiltonian (1) describes the electrons in the two half planes. To describe these 2DEGs, we discretize the space and use a tight binding model on a square lattice,

\[
\hat{H}_{\text{2DEG}} = \sum_{\gamma\sigma} E_{\gamma\sigma} c_{\gamma\sigma}^\dagger c_{\gamma\sigma} - \sum_{n,m,\sigma} t_{nm} c_{\gamma\sigma}^\dagger c_{\gamma\sigma} + \text{H.c.}
\]

where \( c_{\gamma\sigma}^\dagger \) creates an electron with spin \( \sigma \) at site \( \gamma \), and \( t_{nm} \) is the hopping integral between sites \( n \) and \( m \).

![FIG. 1 (color online). Schematic representation of the proposed device. Both QDs are setup to have an odd number of electrons—this is controlled by the gate voltages \( V_1 \) and \( V_2 \). The QDs’ spins are then coupled through a RKKY-like interaction mediated by the electrons in the two 2DEGs. This interaction can be tuned by a perpendicular magnetic field \( B_z \), being maximum when the cyclotron orbit matches the distance between the two QDs. A third (optional) gate \( V_3 \) can be used to interrupt one of the electrons’ path and cancel out the interaction.](image-url)
\( n = (n_x, n_y) \) of the upper (\( \gamma = 1 \)) and lower (\( \gamma = 2 \)) half planes. The hopping matrix element \( t_{nm} \) connects nearest neighbors and includes the effect of the diamagnetic coupling through the Peierls substitution. To avoid any zone boundary effects we take a lattice parameter \( d_0 = 5 \) nm, much smaller than the characteristic Fermi wavelength (\( \lambda_F \approx 50 \) nm). We use the Landau gauge for which \( t_{n(a+b)} = e^{-i n_2 \phi / \hbar} \) and \( t_{n(a+b)} = t \), where \( \phi = a_0 B_z \) is the magnetic flux per plaquette, \( \phi_0 = \hbar c / e \) is the flux quantum, and \( t = \hbar^2 / 2 m^* a_0^2 \) with \( m^* \) the carriers’ effective mass. The third term in the Hamiltonian describes the coupling between the QD and the 2DEGs

\[
\hat{H}_T = - \sum_{\alpha, \gamma} t_{\alpha \gamma} (c^\dagger_{\alpha \sigma} a_{\alpha \sigma} + d^\dagger_{\alpha \sigma} c_{\gamma \sigma}),
\]

where \( c^\dagger_{\alpha \sigma} = N^{-1/2} \sum_{\gamma} c^\dagger_{\gamma \sigma} \) creates an electron at the half plane \( \gamma \) in a linear combination of \( N_0 \) sites next to the QD \( \alpha \). We are interested in the range of magnetic fields that produce a cyclotron radius \( r_c \), of the order of the interdot distance \( R \), defined as the average distance between the sites connected to dot 1 and dot 2. For these small fields, the Zeeman splitting can be neglected, restoring the spin rotational symmetry. In what follows we take \( E_{a1} = E_{a2} \). We assume that the QDs are gated to be in the strong Coulomb blockade regime, \( U + E_{c} \approx - E_{a} \), so that the charge fluctuations can be eliminated by the Schrieffer-Wolf transformation [16]. The spin dynamics is then described by a Kondo Hamiltonian where \( \hat{H}_{QD} + \hat{H}_T \) is replaced by [17]

\[
\hat{H}_K = \sum_{\alpha} J_{\alpha} \hat{S}_{\alpha} \cdot (c^\dagger_{1\alpha \sigma} + c^\dagger_{2\alpha \sigma}) \frac{\hat{S}_{\sigma \sigma'}}{2} (c_{1\alpha \sigma} + c_{2\alpha \sigma}),
\]

where \( \hat{S}_{\alpha} \) is the spin operator associated with the QD \( \alpha \) and

\[
J_{\alpha} = 2|t_{\alpha}|^2 \frac{1}{U_{\alpha} + E_{a}} - \frac{1}{E_{a}} \approx \frac{8|t_{\alpha}|^2}{U_{\alpha}} = \frac{\Gamma_{\alpha}}{U_{\alpha}} 4 \frac{\Gamma_{\alpha}}{\pi \rho},
\]

with \( \Gamma_{\alpha} \) the level width and \( \rho \) the local density of states per spin at \( E_F \). \( \Gamma_{\alpha} \) and \( U_{\alpha} \) can be measured by transport experiments [18,19]. For simplicity, we take \( t_{\alpha \gamma} \equiv t_{\alpha} \) and neglect a potential scattering term which is not relevant for the present work [17]. Finally, usual perturbative procedures give an effective exchange interaction between the QDs’ spins mediated by electrons in the 2DEG. The interdot exchange Hamiltonian \( \hat{H}_J \) contains the nonlocal susceptibility which can be written in terms of one-particle propagators. With a negligible Zeeman splitting, the propagators are spin independent and the Hamiltonian reduces to

\[
\hat{H}_J = J \hat{S}_1 \cdot \hat{S}_2
\]

with

\[
J = - \frac{J_1 J_2}{2\pi} \int d\omega f(\omega) \text{Im}[G_1(1, 2) G_1(2, 1)],
\]

where \( \text{Im} \) denotes the imaginary part and \( G_\sigma(\alpha, \alpha') = \langle \langle (c^\dagger_{1\alpha \sigma} + c^\dagger_{2\alpha \sigma}, (c^\dagger_{1\alpha' \sigma} + c^\dagger_{2\alpha' \sigma})) \rangle \rangle_{\omega+i\eta} \) is the Fourier transform of the retarded Green function [20] and \( f(\omega) \) is the Fermi function. In the following we take \( k_B T \ll J \) in this regime \( f(\omega) \approx \Theta(E_F - \omega) \). To lowest order in \( J_{\alpha} \), the one-particle propagators are calculated with \( \hat{H}_{2DEG} \) only. Then, we have

\[
G_\sigma(\alpha, \alpha') = g_{1\sigma}(\alpha, \alpha') + g_{2\sigma}(\alpha, \alpha'),
\]

with \( g_{\gamma\sigma}(\alpha, \alpha') = \langle \langle c^\dagger_{\gamma \sigma} c_{\gamma \sigma'} \rangle \rangle_{\omega+i\eta} \). Equation (7) makes evident that, in terms of Feynman diagrams, the effective interaction—or the nonlocal susceptibility—is a bubble diagram with a propagator from dot 1 to dot 2 times a propagator from dot 2 to dot 1. In the following, we calculate these propagators numerically (see Ref. [13]).

The field \( B_z \) creates edge states that propagate in opposite directions on opposite sides of the QDs (see Fig. 1). This generates a right-left asymmetry on each half plane, i.e., \( g_{\gamma \sigma}(1, 2) \neq g_{\gamma \sigma}(2, 1) \). In general, with \( B_z \neq 0 \), one of these propagators is very small. In fact, with a single half plane, the backscattering of the edge states is strongly suppressed and they do not contribute to the nonlocal susceptibility. In the proposed geometry, however, tunneling between the two half planes described by Hamiltonian (5) creates a channel for backscattering and the product \( g_{1\sigma}(1, 2) g_{2\sigma}(2, 1) \) does contribute to the effective interdot coupling. For this reason, an effective nonvanishing and controllable spin-spin interaction requires a configuration with the two half planes.

Results for the exchange coupling \( J \) are shown in Fig. 2 and 3 normalized by \( J_0 = J(R \sim \lambda_F, B_z = 0) \). We used parameters typical of GaAs systems—an electron density of \( n = 1.5 \times 10^{11} \) cm\(^{-2} \) that corresponds to a Fermi energy \( E_F = 5 \) meV measured from the bottom of the conduction band and a Fermi momentum \( k_F \approx 0.1 \) nm\(^{-1} \). Figure 2 shows \( J \) as a function of \( B_z \) for interdot distances \( R = 0.5, 1, \) and \( 1.5 \) \( \mu \)m. In all cases, the interaction presents large oscillations whenever the magnetic field is such that twice the cyclotron radius \( r_c = \hbar k_F / e B_z \), is commensurable with \( R \). We refer to these fields as the focusing fields since in this situation electrons that interact with one dot are focused into the other by the action of the field. As \( R \) increases the characteristic period of the oscillations and their amplitude decreases (the nature of these oscillations is discussed below). The exchange coupling \( J \) as a function of the interdot distance is shown in Fig. 3 for fixed fields. For \( B_z = 0 \) the dominant contribution is \( J \propto \cos(2k_F R) / (k_F R) \), in contrast to the conventional 2D behavior \( J \propto \cos(2k_F R) / (k_F R)^2 \). The power law decay \( R^{-4} \) is due to the structure of the states near the edge of the 2DEG. Namely, \( \rho \) depends linearly on energy (it is constant in bulk). Since in a semiclassical picture the contribution to the propagator Eq. (8) arises only from the classical trajectories near the boundary, one could argue that the effective density is \( \rho(\epsilon) \approx \epsilon \), so this case “mimics” a 4D one (thus the \( R^{-4} \) decay). This is confirmed by both a quantum and a semiclassical analytical calculation.

For \( B_z \neq 0 \), a large amplification of \( J \) is observed for distances such that \( R = n 2 R_c \), where \( n \) is an integer. Comparison of Figs. 3(a) and 3(b) shows that at distances...
of the order of 0.5 μm, an increase of the field from zero to its focusing value increases the coupling by more than 2 orders of magnitude (notice \( J/J_0 \sim 1 \) for \( n = 1 \)). Additionally, the sign of the interaction is controlled by finely tuning the magnetic field around the focusing value [see arrows in Fig. 2(a)]. Note that a larger \( N_0 \) (tunneling region) enhances the effect. This is due to a reduction of diffraction effects, which leads to a better definition of the classical cyclotron orbit [13].

All these results can be put together in a single density plot as shown in Fig. 4. The lines are hyperbolas corresponding to different focusing fields

\[
B_z^{(n)} = 2n(\hbar c k_F/eR) = 2n \phi_0/\lambda_F R. \quad (9)
\]

Along these lines, i.e., when field and distance are simultaneously varied to keep the focusing condition, the exchange coupling oscillates with large amplitude. A simple fitting of the numerical results shows that the dominant contribution to \( J \) along the hyperbolas is given by

\[
J \propto \frac{\cos(k_F \pi R/2 + \varphi)}{R^2}. \quad (10)
\]

Note that the interaction decays as \( R^{-2} \) as in the usual 2D case—this is consistent with the semiclassical picture since now the classical trajectories are not restricted to be close to the boundary. The argument of the cosine can be written as \( k_F R_{\text{eff}} \), where \( R_{\text{eff}} = \pi R/2 \) is the length of the classical trajectory (see Fig. 1). The period of the oscillations is then equal to the period at which the Landau levels cross \( E_F \). In fact, we have \( k_F R_{\text{eff}} = 2\pi R_{\text{eff}}/\lambda_F = 2\pi E_F/\hbar c \). These oscillations of the nonlocal susceptibility are the analog of the de Haas–van Alphen oscillations of the magnetization. The complex oscillating pattern observed in Fig. 2 corresponds to a cut in Fig. 4 along a vertical line. The inset in Fig. 4 shows the coupling \( J \) when the field and the interdot distance are varied along the first hyperbola (\( n = 1 \)). The solid line is a fitting to Eq. (10).

So far we have presented results obtained by fixing the chemical potential. However, in a 2DEG with a fixed charge density, \( E_F \) is pinned at the energy of the partially filled Landau level and presents periodic jumps when plotted as a function of \( 1/B_z \). Figure 5 shows the coupling \( J \) versus the external field for constant electron density at the bulk of the 2DEG. Now again, the spin-spin interaction presents a large enhancement at the focusing fields. There are, however, some differences with the previous case: as the external field is varied around the focusing values,
the sign of the interdot interactions tends to be preserved. Both ferromagnetic and antiferromagnetic couplings are obtained. The dominant sign of \( J \) at the different focusing field depends on the parameters, in particular, on the particle density. The jumps of \( E_p \) overestimate some charge redistribution at the edges. Including the electron-electron interactions in a self-consistent approximation would tend to preserve local charge neutrality. This may generate an intermediate situation where the effective coupling changes sign as \( B_r \) sweeps the focusing values.

It is worth pointing out that in the presence of a magnetic field, the exchange interaction between two spins in the bulk of a 2DEG also shows some structure: there is a small enhancement of \( J \) for \( R \approx 2r_c \) while it decays exponentially for \( R \gg 2r_c \). In the proposed geometry, however, the focusing effect produces an amplification of \( J \) much larger than what is obtained in an homogeneous 2DEG. Moreover, with present technologies, it is possible to build a device like the one schematically shown in Fig. 1, where the contacts used to control the QD parameters (\( V_{\text{side}} \) like the one schematically shown in Fig. 1, where the central gate with the same or different interdot distances, interrupting the particle propagation in one of the 2DEG. When the cyclotron radius \( r_c \) becomes commensurable with the interdot distance \( R \) there is a large amplification of the interdot interaction. This condition, \( 2n r_c = R \), defines the focusing fields. As the external field is varied around this value, the enhanced interaction changes sign allowing for a fine tuning of a ferromagnetic or an antiferromagnetic coupling.

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In summary, we showed that two QDs at the edge of a 2DEG interact with an exchange coupling \( J \) that can be controlled with a small magnetic field perpendicular to the 2DEG. When the cyclotron radius \( r_c \) becomes commensurable with the interdot distance \( R \) there is a large amplification of the interdot interaction. This condition, \( 2n r_c = R \), defines the focusing fields. As the external field is varied around this value, the enhanced interaction changes sign allowing for a fine tuning of a ferromagnetic or an antiferromagnetic coupling.