Coherent control and measurement of quantum spins are at the heart of new technologies with great potential value for information processing. This has lead to a great activity in the field of quantum spin control in solid state devices. Since the seminal work by Loss and DiVincenzo, the exchange gate is founded on the Heisenberg exchange mechanism relies on the interplay between transverse electron focusing and spin-orbit coupling allows to dynamically change the symmetry of the effective spin-spin Hamiltonian. The question then is: Is it possible to engineer a predefined spin-spin interaction between QDs and then change its magnitude, sign and symmetry with a negligible impact on the internal structure of the dots?

Here we show how to design a Heisenberg or an Ising-like interaction of the desired magnitude and sign of the coupling constant and then dynamically change one into the other by controlling a small magnetic field—the control mechanism relies on the interplay between transverse electron focusing and spin-orbit (SO) coupling. This opens up the possibility to manipulate spin-spin Hamiltonians in solid state devices as it is done today with nuclear magnetic resonance techniques in molecules.

The setup consists of two QDs at the edges of two-dimensional electron gases (2DEG) as schematically shown in Fig. 1(a), with an interdot distance \( d \) of the order of 1 \( \mu m \). In the Coulomb blockade regime, the QDs can be gated to have an odd number of electrons so that they behave as magnetic objects. In what follows we describe them as localized \( \frac{1}{2} \) spins. The virtual tunneling of electrons between the dots and the 2DEG leads to a Kondo coupling between the localized spins \( S_i \) and the 2DEG spins described by

\[
\hat{H}_K = \sum_{i,\eta,\gamma} J_i S_i \cdot \psi^\dagger_{\eta\sigma}(R_i) \frac{\sigma_{\eta\gamma}}{2} \psi_{\eta\gamma}(R_i),
\]

where \( i = 1,2 \) indicates the left and right QD, respectively. \( \psi_{\eta\sigma}(R_i) \) creates an electron with spin \( \sigma \) in a Wannier-like orbital centered around the coordinate \( R_i \) next to the \( i \)th QD at the upper (\( \eta = 1 \)) or lower (\( \eta = 2 \)) plane. The spacial extension of the Wannier orbital depends on the opening of the QDs. This coupling leads to a Ruderman–Kittel–Kasuya–Yosida (RKKY)-like interaction between the QDs spins,

\[
\hat{H}_J = -\frac{J_1 J_2}{4 \pi} \text{Im} \int d\omega f(\omega) \text{Tr}[\sigma_1 G(1,2) \sigma_2 G(2,1)],
\]

where \( f(\omega) \) is the Fermi function and the \( 2 \times 2 \) matrix \( G(i,j) \) is the Fourier transform of the retarded electron propagator whose elements are \( G_{\alpha\beta}(i,j,t,t') = -i \theta(t-t') \times \sum_{\eta,\gamma} \langle \psi_{\eta\alpha}(R_i,t) | \psi_{\gamma\beta}(R_j,t') \rangle \). When the electron’s spin is conserved along the electron propagation between QDs, \( G(i,j) \) is diagonal in the spin index and the spin-spin Hamiltonian (2) reduces to \( \hat{H}_J = J S_1 \cdot S_2 \) with the usual RKKY-like exchange \( J = J_1 J_2 / 2 \pi \text{Im} \int d\omega f(\omega) G(1,2) G_{11}(2,1) \).

The presence of a small magnetic field \( B_z \) perpendicular to the 2DEG creates edge states that dominate the electron scattering from objects placed at the 2DEG edges. The interaction between QDs is then mediated by these edge states and the propagators are mainly due to the semiclassical orbits shown in Figs. 1(b) and 1(c). Due to the chiral nature of

![FIG. 1.](image-url)
these orbits, the intra-plane scattering, described by the terms in Eq. (1) with $\eta=\eta'$, give forward scattering while the inter-plane terms (with $\eta \neq \eta'$) describe the backward scattering. Only the inter-plane backward scattering processes contribute to the effective interaction. In other words, each propagator in Eq. (2) is due to contributions from one plane only. As the external field increases, the cyclotron radii of these orbits decrease: $r_{ci}=\hbar k_c/eB_i$ with $k$ the electron wave vector. The focusing fields are those for which the interdot distance $d$ is commensurate with the cyclotron radius $r_{ci}$ of electrons at the Fermi energy ($E_F$), that is $d=2n r_{ci} = 2n \hbar k_c/eB_i$ with $n$ an integer number. At the focusing fields, the electrons at the Fermi level scattered by one QD are focused onto the other at the Fermi energy around the first focusing condition the propagators numerical result is shown in Fig. 1(d) where, for the sake of comparison with the conventional RKKY interaction, the exchange integral $J$ is plotted as a function of the interdot distance for a fixed magnetic field. These results were obtained using a finite differences technique$^{13,16}$ for a system with an effective electronic mass $m'=0.067 m$ and $E_F=5$ meV, corresponding to an electron density of $1.5 \times 10^{11}$/cm$^2$. With these parameters, the focusing amplification of the exchange integral is clearly observed. In the semiclassical picture, the first focusing condition ($n=1$) corresponds to a direct propagation of the electrons from one QD to the other; in the second one ($n=2$) the electron bounces once at the 2DEG edge. For interdot distances of the order of 1 $\mu$m, the magnetic fields for the first focusing conditions ($n=1$ or 2) are small and neglecting the Zeeman spin splitting due to the external field is a good approximation. It is worth mentioning that electron focusing in similar geometries is clearly observed in transport experiments.$^7,11,19$

In systems with strong spin-orbit (SO) coupling, new effects arise. We consider a Rashba-SO interaction in the 2DEG.$^{20,21}$ This interaction is due to the inversion asymmetry of the confining potential and it is described by the Hamiltonian $H_{SO}=\alpha S \cdot (\mathbf{p} \times \mathbf{A})$, where $p_\gamma=p_\gamma(e/c)A_\gamma$ with $p_\gamma$ and $A_\gamma$ being the $\gamma$ component of the momentun and vector potential, respectively, and $\sigma_\gamma$ the spin operators. The SO coupling acts as a strong in-plane magnetic field proportional to the momentum. This breaks the spin degeneracy leading to two different conduction bands.$^{20}$ In the presence of a small magnetic field perpendicular to the gas plane, each band leads to a different cyclotron radius. These two radii manifest as two distinct focusing fields for the first ($n=1$) focusing condition.$^{13}$ This splitting has been observed by Rokhinson et al.$^{14}$ in a $p$-doped GaAs/AlGaAs heterostructure. The spin texture of the orbits is such that, for small fields (large cyclotron radii), the electron’s spin adiabatically rotates along the semiclassical orbit, being perpendicular to the momentum, as schematically shown in Fig. 2. In order to describe the magnetic scattering of electrons in this case, it is convenient to quantize the spin along the $x$ axis. Then, around the first focusing condition the propagators $G_{\uparrow\downarrow}(i,j)$ and $G_{\downarrow\uparrow}(i,j)$ dominate the interdot coupling; here the spin index $\uparrow$ indicates the spin projection. The interdot interaction is then approximately given by an Ising term $\hat{H}_J = J_{x} S_1 \cdot S_2$, with coupling constant given by

$$J_{x}=\frac{J_2 J_3}{4\pi} \Im \int d\omega (\omega G_{\uparrow\downarrow}(i,j)G_{\downarrow\uparrow}(j,i)),$$

where $i=1$ and $j=2$ or $i=2$ and $j=1$ depending on which cyclotron radius contributes to the focusing. This result can be visualized in terms of the semiclassical trajectories shown in Figs. 2(a) and 2(b): for a SO coupling strong enough to split the focusing condition, the inter-plane spin-flip back-scattering mixes the two cyclotron radii living the electron out of the focusing condition. Thus, these processes cannot contribute to the coupling. The interdot interaction is then due to non-spin-flip processes of electrons that are backscattered. This defines the symmetry axis of the Ising interaction.

At the second focusing condition, the system operates in a different way [see Fig. 2(c)]. There are two important effects to consider: (i) the orbits with different cyclotron radii are mixed at the bouncing point due to spin conservation, and (ii) along the trajectories from one QD to the other the electron’s spin completes a $2\pi$ rotation. Then, the two orbits contribute to the exchange integral and $G_{\uparrow\downarrow}(i,j)$ and $G_{\downarrow\uparrow}(i,j)$ dominate the spin-spin coupling. In this way the rotational symmetric Heisenberg coupling is recovered.

For arbitrary external field, Hamiltonian (2) can be written as fully anisotropic Heisenberg model plus a Dzyaloshinski–Moriya term

$$\hat{H}_J = \sum_{\gamma} J_{\gamma} S_1 \cdot S_2 + \mathbf{\beta} \cdot (\mathbf{S}_1 \times \mathbf{S}_2),$$

where $\mathbf{\beta}=(0,\beta_0,0)$. Hamiltonian (4) is a particular case of a more general Hamiltonian including SO effects.$^{22-24}$ In our case, due to the symmetry of our geometry, we only have four independent parameters: $J_{\gamma\gamma}$ with $\gamma=x,y,z$, and $\beta_0$. The numerical results for these coupling constants are shown in Fig. 2(d). As argued above, around the first focusing condition the system behaves as an Ising-like model: the dominant couplings in these peaks depend on both $B_z$ and $E_F$. At the second focusing condition the system behaves as an isotropic Heisenberg model ($J_{xx}=J_{yy}=J_{zz}$) with a small anisotropic correction ([$\left| \beta_0 / J_{xx}\right|<1$). Figures 3(a) and 3(b) show the dominant couplings $J_{xx}$ and $J_{zz}$, respectively, as a function of $B_z$ and $d$. The magnetic field not only can turn on and off each coupling but a fine tune around the focusing fields

\begin{figure}[h]
\centering
\includegraphics[width=0.9\textwidth]{fig2.png}
\caption{Schematics of the first focusing condition for the smallest (a) and largest (b) cyclotron radius and of the second focusing condition (c) in the presence of SO coupling. The arrows indicate the spin orientation. In (d) the four coupling constants of Hamiltonian (4) are shown as a function of $d$. Here $\alpha=15$ meV nm, and the other parameters as in Fig. 1.}
\end{figure}
can change their sign, too [see Figs. 3(c) and 3(d)].

There are a variety of systems that are potentially appropriate to observe these effects. While in $n$-doped GaAs/AlGaAs heterostructures the SO is small, in systems like $p$-doped GaAs/AlGaAs or InGaAs heterostructures it is large. The nature of the SO effect depends on the system. Effects like the ones discussed here are also present in systems with Dresselhaus SO coupling. Furthermore, the external control of the relative magnitude of both contributions to the SO coupling \(^1\) could allow the control of the quantization axis of the Ising-like interaction.

In summary, the interplay between transverse electron focusing and spin-orbit interaction gives a unique opportunity to control and tune the spin-spin interaction between QDs without inducing big changes in their internal structure. When the SO coupling is large, it leads to a spin-dependent focusing condition (for $n=1$), resulting in a highly anisotropic Ising-like interaction. However, by doubling the external field a Heisenberg gate with a small correction of the Dzyaloshinskii–Moriya type is recovered. In the context of quantum computing, there are strategies to eliminate or control these small corrections to the Heisenberg gate. \(^2\)–\(^2\)\(^7\)\(^\) Our setup can be extended to three or more QDs in a linear array. An array with different interdot distances and with extra gates used to blockade the focusing may be used to independently control the interdot interactions. Such array would also allow to study the interplay between different interactions of three or more localized spins and the Kondo screening.\(^2\)\(^8\)–\(^3\)\(^0\)

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