Rough interfaces
and
Elastic lines in disordered systems

1-Basic concepts
- Ferromagnetic domain wall
- Simplified models, interactions
- Random walk
- Fluctuations: Roughness

2-Theoretical formulation
- Correlation functions
- Family-Vicsek scaling
- Edwards-Wilkinson equation

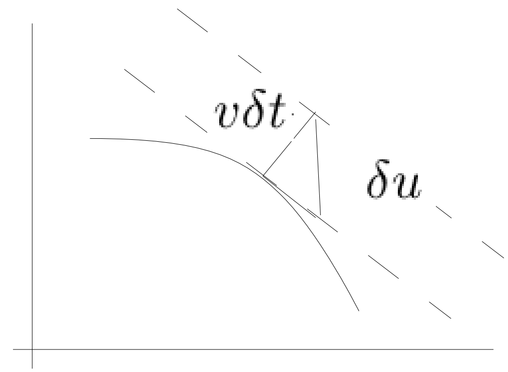
3-Universality
- Kardar-Parisi-Zhang equation
- Mullins-Herring equation
- Universality classes
- Distribution functions
- Anomalous scaling

4-Disordered media
- Quenched disorder
- Directed polymer in random media
- Ground state and transfer matrix
- Energy fluctuations
- Thermal effects

5-Depinning transition
- Experimental results
- Critical phenomena analogy
- Correlation lengths
- Dynamic regimes, thermal effects

Kardar-Parisi-Zhang equation

first non-linear correction
for lateral growing



$$\delta u \quad = \quad \sqrt{(v\delta t)^2 + (v\delta t \partial_z u)^2} = v\delta t\sqrt{1 + (\partial_z u)^2}$$

gradient
expansion

$$\frac{\partial u}{\partial t} \quad = \quad v + \frac{v}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \ldots$$

$$\frac{\partial u(z,t)}{\partial t} = \nu \frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\lambda}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \eta(z,t)$$

Kardar-Parisi-Zhang equation

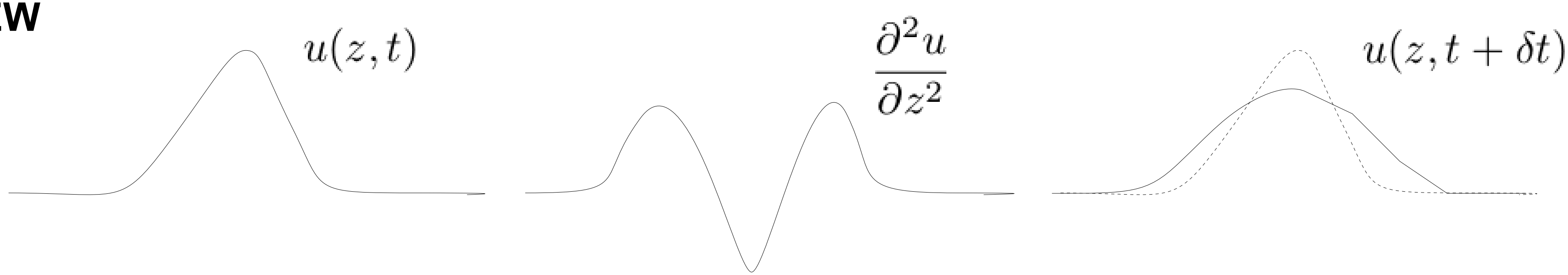
No Hamiltonian approach since the $u \to -u$ symmetry is broken

Kardar, Parisi, Zhang, PRL. **56**, 889, 1986

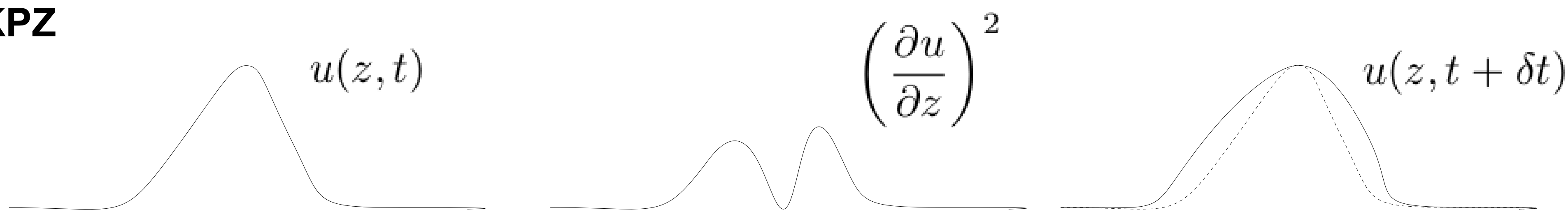
Kardar-Parisi-Zhang equation

$$\frac{\partial u(z,t)}{\partial t} = \nu \frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\lambda}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \eta(z,t)$$

EW



KPZ



Kardar-Parisi-Zhang equation

The KPZ equation has an intrinsic finite velocity contribution

$$
\begin{aligned}
\frac{\partial u(z,t)}{\partial t} &= \nu \frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\lambda}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \eta(z,t) \\
\int_L dz \frac{\partial u(z,t)}{\partial t} &= \nu \int_L dz \frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\lambda}{2}\int_L dz \left(\frac{\partial u}{\partial z}\right)^2 + \int_L dz \eta(z,t) \\
\frac{\partial \overline{u}}{\partial t} &= \frac{\lambda}{2}\int_L dz \left(\frac{\partial u}{\partial z}\right)^2 > 0
\end{aligned}
$$

Kardar-Parisi-Zhang equation

Langevin equation $\partial_t y = G(y) + \eta(t)$

Fokker-Planck equation $\partial_t P(y,t) = \partial_y \left[-G(y)P(y,t) + D/2\partial_y P(y,t)\right]$

$$\partial_t u = \nu \partial_z^2 u + \frac{\lambda}{2} \left(\partial_z u\right)^2 + \eta(z,t)$$

$$\partial_t P[u(z,t)] = \int dz \frac{\delta}{\delta u} \left\{ - \left[\nu \partial_z^2 u + \frac{\lambda}{2} \left(\partial_z u\right)^2 \right] P[u(z,t)] + \frac{D}{2} \frac{\delta}{\delta u} P[u(z,t)] \right\}$$

the stationary solution $\partial_t P_S[u(z,t)] = 0$ becomes

$$P_S[u(z,t)] = \exp\left[\frac{\nu}{D} \int dz \left(\partial_z u\right)^2 \right]$$

this implies that the nonlinear KPZ term is irrelevant at long times and thus that $\alpha_{\mathrm{KPZ}} = 1/2$

Kardar-Parisi-Zhang equation

Galilean invariance:
the KPZ equation is invariant under

$$\begin{cases} z \rightarrow z - \lambda v t \\ u \rightarrow u + v z \\ F \rightarrow F - \lambda v^2/2 \end{cases}$$

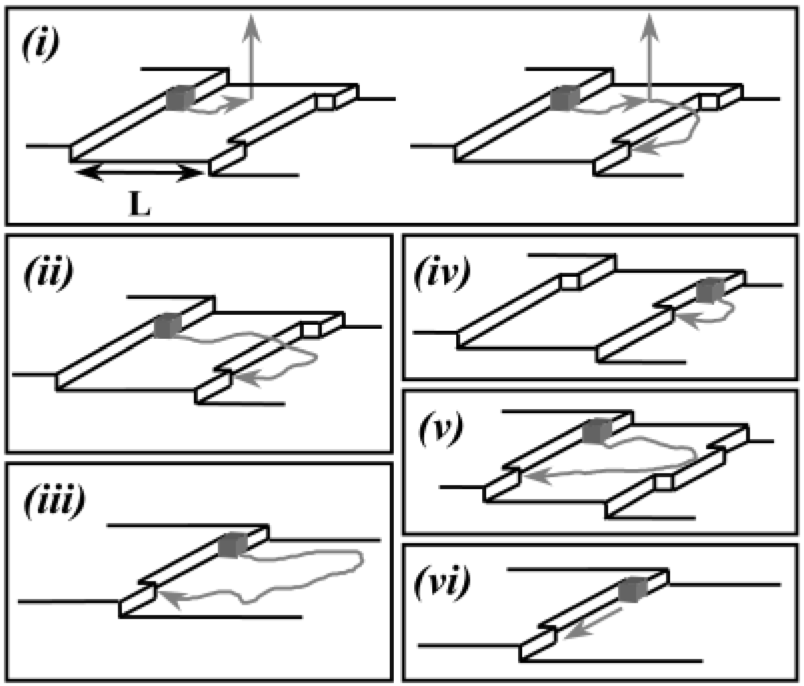
$$\partial_t u = \nu \partial_z^2 u + \frac{\lambda}{2} (\partial_z u)^2 + F + \eta(z,t)$$

this implies that the nonlinear term is invariant under $z \rightarrow z' = bz \quad u \rightarrow u' = b^\alpha u$

therefore the following scaling relations holds: $z + \alpha = 2$

$$\boxed{\alpha_{\mathrm{KPZ}} = 1/2 \quad z_{\mathrm{KPZ}} = 3/2 \quad \beta_{\mathrm{KPZ}} = 1/3}$$

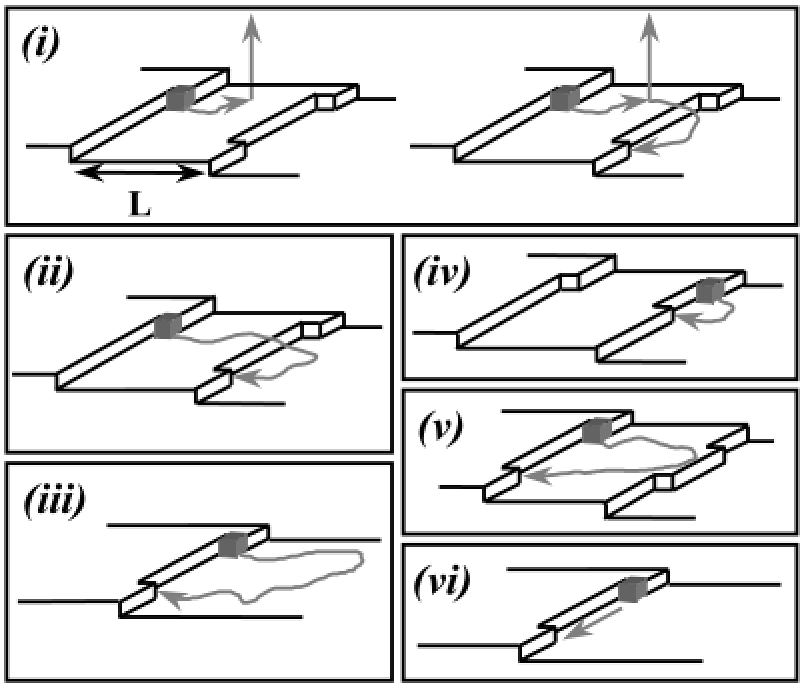
Mullins-Herring equation



$$\gamma \partial_t u = \nabla^2 \frac{\delta \mathcal{H}}{\delta u} + \eta$$

- Cahn-Hilliard model
- Model B
- purely dissipative dynamics for a conserved variable

Mullins-Herring equation



$$\gamma \partial_t u = \nabla^2 \frac{\delta \mathcal{H}}{\delta u} + \eta$$

$$\mathcal{H} = c \int dz \left(\frac{\partial u}{\partial z} \right)^2$$

$$\partial_t u = -\nu \frac{\partial^4 u}{\partial z^4} + \eta$$

$$\partial_t c_n = -\nu q_n^4 c_n + \eta_n$$

$$c_n = \int_0^t e^{-\nu q_n^4 (t-t')} \eta_n(t') dt'$$

$$S_n(t) = \frac{T}{q_n^2} \left(1 - e^{-\nu q_n^4 t} \right) = q_n^{-2} f \left(q_n t^{1/4} \right)$$

$$z_{\mathrm{MH}} = 4 \quad \alpha_{\mathrm{MH}} = 1/2 \quad \beta_{\mathrm{MH}} = 1/8$$

Universality

universality in terms of exponentes:

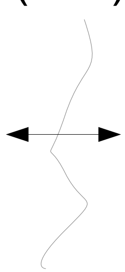
- roughness exponent α
- dynamic exponent z
- growing exponent β

interactions

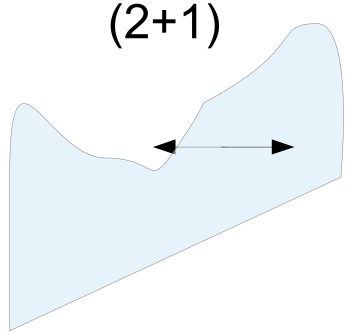
$$\partial_t u = \underbrace{\nu \partial_z^2 u}_{\text{EW}} + \underbrace{(\partial_z u)^2\, \partial_z^2 u}_{\text{anharmonic}} + \underbrace{\frac{\lambda}{2}(\partial_z u)^2}_{\text{KPZ}} + \underbrace{\int \frac{u(z')}{|z - z'|} dz'}_{\text{long-range}} + \underbrace{\eta(z,t)}_{\text{noise}} + \underbrace{\xi(u,z)}_{\text{quenched noise}} + \ldots$$

dimensionality

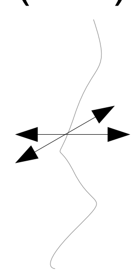
(1+1)



(2+1)

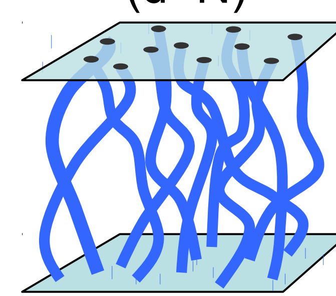


(1+2)



...

(d+N)



vortices (3+2)

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

α : global roughness exponent
α_{loc}: local roughness exponent
α_S : spectral roughness exponent

$$B(r,t) = \int \frac{dq}{\pi}\ [1 - \cos(qr)]\ S(q)$$

The convergence of the integral depends on the value of α_S

$$B(r) \sim r^{2\alpha_{loc}} \sim \begin{cases} r^{2\alpha_S} & \text{for } \alpha_S < 1 \quad \alpha_{\text{loc}} = \alpha_s \\ r^2 & \text{for } \alpha_S > 1 \quad \alpha_{\text{loc}} = 1 \end{cases}$$

$\alpha_S < 1$, $\alpha_{\text{loc}} = \alpha_s$: intrinsic anomalous scaling
$\alpha_S > 1$, $\alpha_{\text{loc}} = 1$: super rough anomalous scaling

López, Rodriguez, Cuerno, PRE, **56**, 3993, 1997

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

General classification for anomalous scaling

$$\begin{cases} \text{if } \alpha_s < 1 \Rightarrow \alpha_{\text{loc}} = \alpha_s & \begin{cases} \alpha_s = \alpha \Rightarrow & \text{Family-Vicsek} \\ \alpha_s \neq \alpha \Rightarrow & \text{intrinsic} \end{cases} \\ \text{if } \alpha_s > 1 \Rightarrow \alpha_{\text{loc}} = 1 & \begin{cases} \alpha_s = \alpha \Rightarrow & \text{super rough} \\ \alpha_s \neq \alpha \Rightarrow & \text{faceted} \end{cases} \end{cases}$$

Ramasco, López, Rodriguez, PRL, **84**, 2199, 2000

Anomalous scaling

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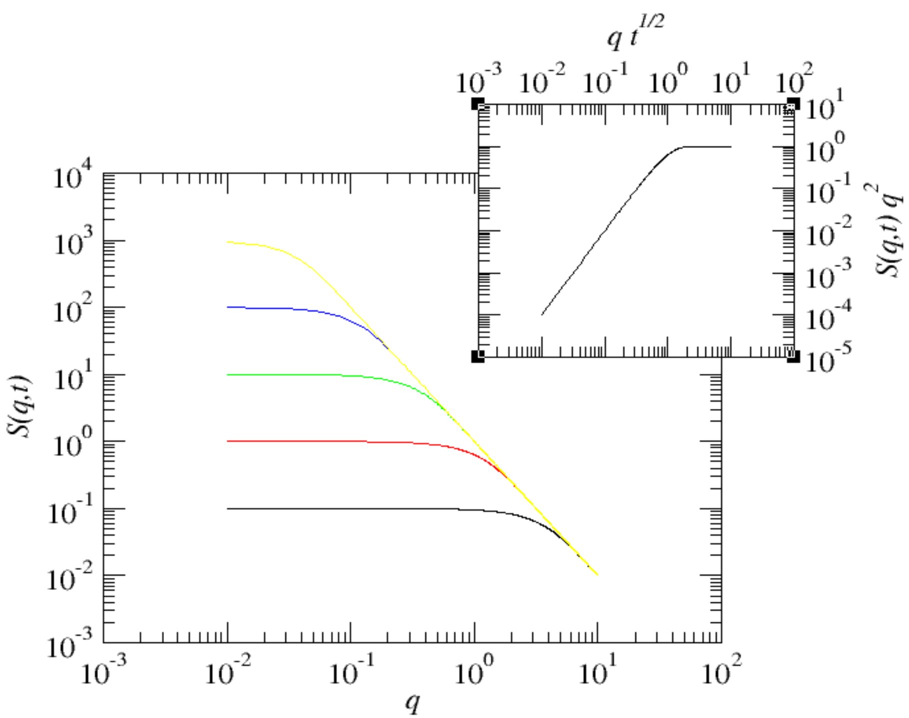

Edwards-Wilkinson equation

$$S(q,t) \sim \begin{cases} t^{(1+2\alpha)/z} & \text{for } q \ll \xi(t)^{-1} \\ q^{-(1+2\alpha)} & \text{for } q \gg \xi(t)^{-1} \end{cases}$$

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

General classification for anomalous scaling

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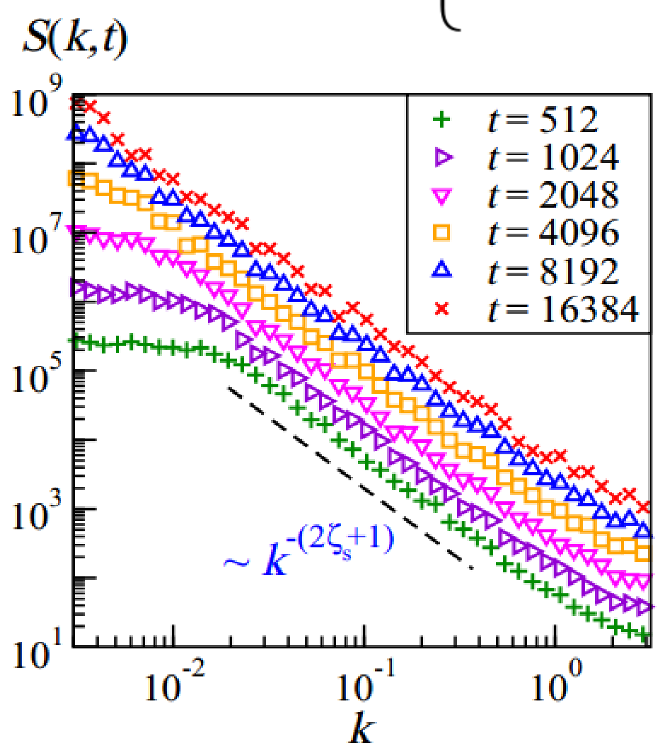
	$\alpha_s \neq \alpha \Rightarrow$	**intrinsic**
if $\alpha_s > 1 \Rightarrow \alpha_{\rm loc} = 1$	$\alpha_s = \alpha \Rightarrow$	super rough
	$\alpha_s \neq \alpha \Rightarrow$	faceted



$$\alpha_S = 0.5$$

ferromagnetic thin film model

$$S(q,t) \sim \begin{cases} t^{(1+2\alpha)/z} & \text{for } qt^{1/z} \ll 1 \\ q^{-(1+2\alpha_S)} t^{2(\alpha-\alpha_S)/z} & \text{for } qt^{1/z} \gg 1 \end{cases}$$

Torres, Buceta, EPJB, **86**, 20, 2013

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

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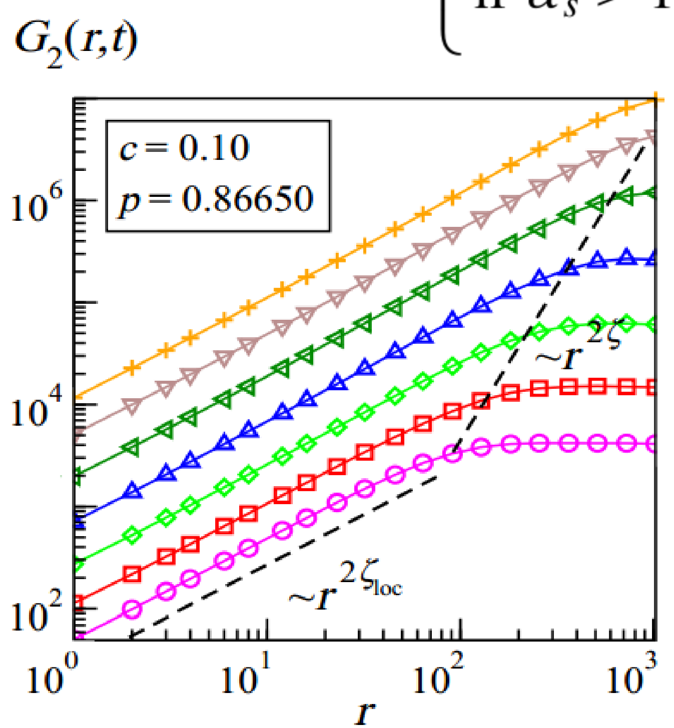

$$\alpha_S = 0.5 \quad \alpha_{loc} = 0.5$$

$$B(r,t) \sim \begin{cases} r^{2\alpha_{loc}} t^{2(\alpha-\alpha_{loc})/z} & \text{for } rt^{-1/z} \ll 1 \\ t^{2\alpha/z} & \text{for } rt^{-1/z} \gg 1 \end{cases}$$

Torres, Buceta, EPJB, **86**, 20, 2013

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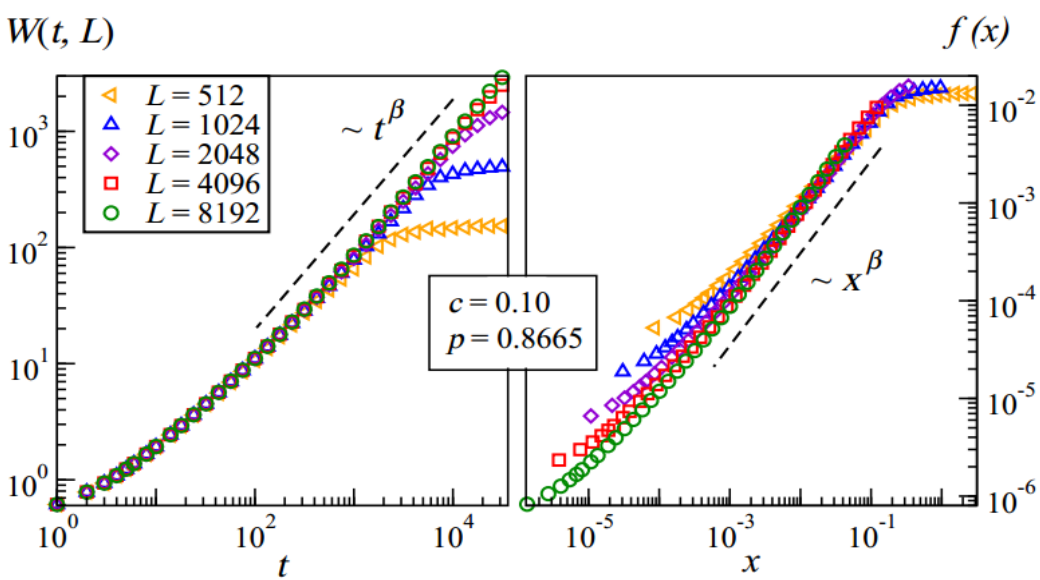
	$\alpha_s \neq \alpha \Rightarrow$	faceted

$$\alpha_S = 0.5 \quad \alpha_{loc} = 0.5 \quad \alpha = 1.5$$



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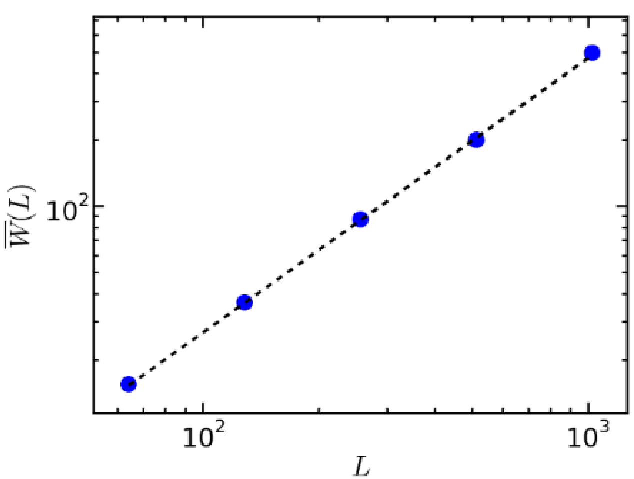
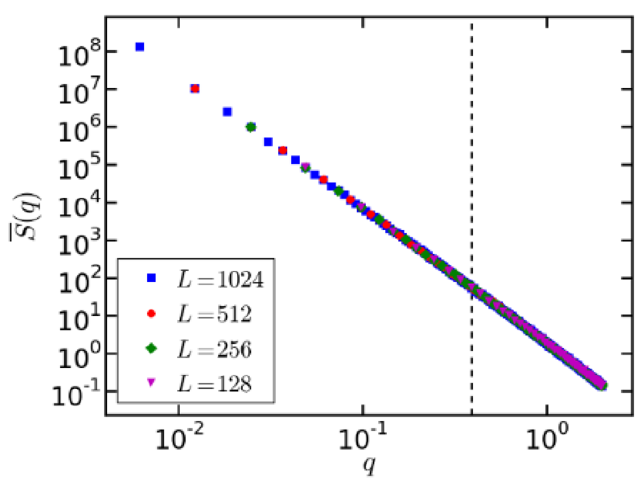
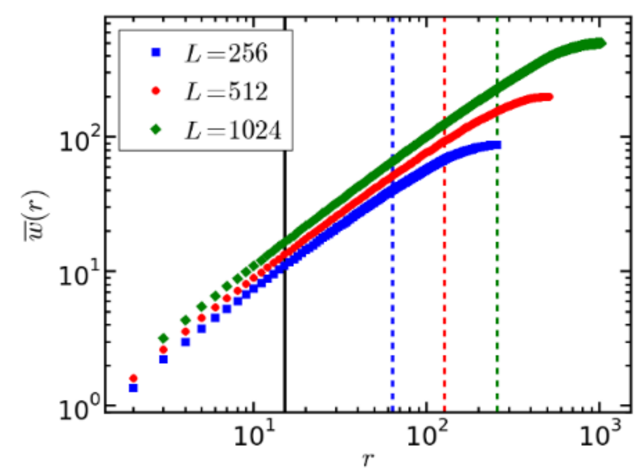
	$\alpha_s \neq \alpha \Rightarrow$	faceted

interface at critical depinning driven quenched EW equation

$$\alpha_S = 1.25 \quad \alpha_{loc} = 1 \quad \alpha = 1.25$$







Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

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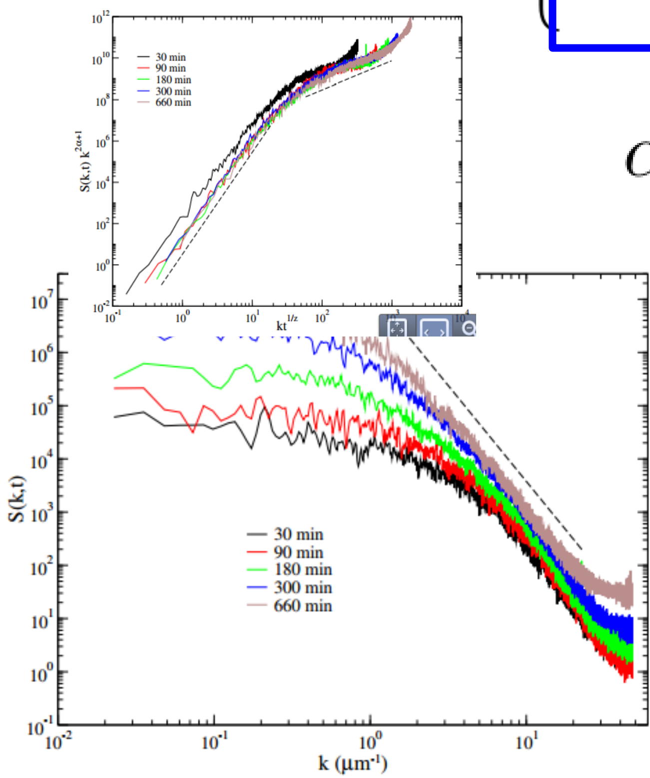
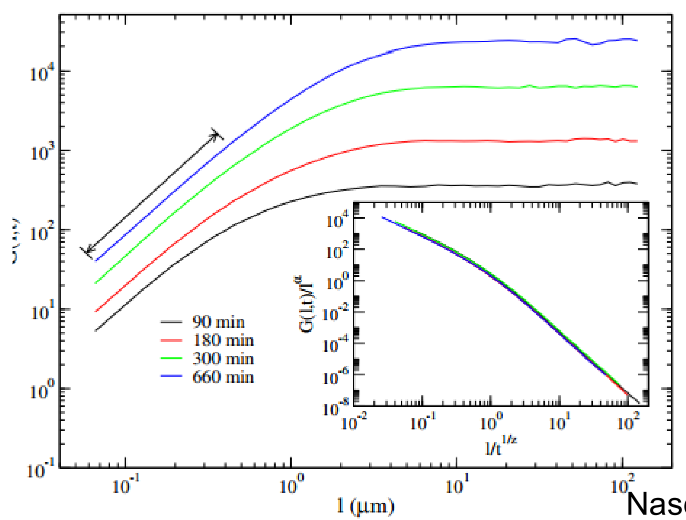
$$\alpha_S = 1.29 \quad \alpha_{loc} = 0.82 \quad \alpha = 1.94$$





epitaxial growth of semiconductor films (2+1)

Nascimiento, Ferreira, Ferreira,EPL, **94**, 68002, 2011