A theory of afterpulse formation in photomultipliers and the prepulse height distribution

This article has been downloaded from IOPscience. Please scroll down to see the full text article.
(http://iopscience.iop.org/0022-3727/6/16/306)
The Table of Contents and more related content is available

Download details:
IP Address: 200.0.233.51
The article was downloaded on 15/01/2010 at 17:37

Please note that terms and conditions apply.
A theory of afterpulse formation in photomultipliers and the prepulse height distribution

P B Coates
Division of Quantum Metrology, National Physical Laboratory, Teddington, Middx, TW11 0LW

Received 6 June 1973

Abstract. The amplitude distribution of prepulses—those pulses preceding ion afterpulses in photomultipliers—depends upon the point of origin of the afterpulse. This fact provides a method of confirming the discovery that, in RCA types 8850 and 8852, afterpulses were formed throughout the photomultiplier. A simple theory of afterpulse formation in the region between the cathode and the first dynode is used to derive the prepulse height distribution.

1. Introduction

The formation of an afterpulse in a photomultiplier is characterized by the appearance at the anode of two pulses separated by a time interval of the order of 1 μs. While the generating pulse, or prepulse, is often indistinguishable from the mass of single electron pulses, the afterpulse is usually noticeably larger, and may correspond in amplitude to the release of up to 20 electrons from the photocathode. Interest in this subject has previously centred on the well-defined values exhibited by the time interval, and on the afterpulse height distribution. As far as the author is aware, no measurements have yet been made on the characteristics of the prepulse, but it is the purpose of this paper to indicate that these may give some information on the mechanism of afterpulse formation.

It has been tacitly assumed in previous work that the prepulse height distribution \( F(h) \) was not significantly different from the single electron response \( S(h) \) of the photomultiplier. However, the first stage in the formation of an afterpulse involves the ionization of a gaseous impurity \( A \) by electron collision, which may be represented by the equation

\[
A + e^- \rightarrow A^+ + 2e^-.
\]

The extra electron added to the number already present in the prepulse has little effect if ionization takes place after the first dynode, especially in those tubes in which the first dynode has high gain. For such events \( F(h) \) should therefore be very similar to \( S(h) \). However, for afterpulses formed between the cathode and the first dynode, the number of electrons is doubled and the prepulse height distribution should be significantly different. (To avoid repetition, afterpulses formed before the first dynode will be referred to as group A afterpulses, those formed after as group B.)

Since \( F(h) \) depends upon the origin of the afterpulses in the photomultiplier, it may be used to test the discovery (Coates 1973b) that, in the RCA type 8850 and 8852 photomultipliers, afterpulses were formed throughout the multiplier, and not solely in the...
The prepulse height distribution cathode/first-dynode region, as suggested by Morton et al (1967). In addition, although it is not yet possible to provide a general quantitative theory for the formation of afterpulses, a simple theory will be given for group A pulses which illustrates the mechanism of afterpulse formation, and may be used to derive \( F(h) \).

2. Experimental technique

The apparatus used to measure \( F(h) \) was similar to that constructed to observe the distribution of the time intervals \( \tau \) between the prepulse and the afterpulse (Coates 1973b). In this paper it was shown that \( \tau \) not only characterized the nature of the ion producing the afterpulse, but also its origin in the photomultiplier. By selecting those pulses from the time-to-amplitude converter (TAC) with amplitudes in a particular range, with a window discriminator, it was therefore possible to measure \( F(h) \) for group A and group B afterpulses separately. In the first case, the efficiency of separation (figure 1) could be improved considerably by adjusting the potential divider network (Coates 1973b). By illuminating the centre of the photocathode at a constant intensity, and keeping the discriminator level in the afterpulse or stop channel fixed, the integral of \( F(h) \) could be obtained by counting the number of pulses passed by the window discriminator in a given time as a function of the discriminator level in the prepulse or start channel. However, the accuracy achieved was low, since the fraction of afterpulses was less than 1% of the single electron count rate. Slow variations in the intensity of illumination, the

![Figure 1. Time distribution for the 8850 photomultiplier, showing time gates \( T_2 - T_1 \) determined by the window discriminator.](image-url)
gain of the photomultiplier and the zero setting of the start discriminator also gave
rise to errors if the duration of the experiment was extended. To reduce these, the experi-
ment was modified to measure the ratio \( R(h) \) of the number of afterpulse events to the
total number of intervals analysed by the TAC (obtained from the ‘true start’ output of
this unit) above a discriminator setting \( h \). At the start of an experiment, the counting
efficiency

\[
\alpha(h) = \int_{h}^{\infty} S(h') \, dh'
\]

was determined to better than 1% accuracy. Since the single electron count rate was
about \( 10^4 \, \text{s}^{-1} \), this determination could be performed without introducing errors from
the sources given above. At intervals during the measurement of \( R(h) \), \( \alpha(h) \) was checked
at a few points to measure changes in the zero pulse height discriminator setting and the
effective gain. Corrections could then be applied to the measured values of \( R(h) \), so
that this was obtained on the same scale as \( \alpha(h) \). The integral of the prepulse height
distribution was obtained from

\[
\int_{h}^{\infty} F(h') \, dh' = \alpha(h) \, R(h).
\]

This and \( \alpha(h) \) were then differentiated numerically to obtain \( F(h) \) and \( S(h) \) respectively.
This method had two additional advantages: it eliminated the estimation of pile-up
errors from the conversion time of the TAC, and it gave the single electron response
\( S(h) \) on the same scale as \( \alpha(h) \). This was required for the comparison of the measured
and calculated distributions.

3. Theory

Since the prepulse height distribution \( F(h) \) for group B afterpulses was expected to be
very similar to the single electron response, it was only necessary to calculate \( F(h) \) for
group A afterpulses. This was fortunate, since the complexity of the electric field distri-
bution after the first dynode precludes even the most approximate theories of afterpulse
formation there (Coates 1973b). The electric field between the cathode and the first
dynode is not as complex, and the majority of the ions formed in this region reach the
cathode.

Because pulse amplitude distributions in GaP photomultipliers are determined by the
characteristics of the first dynode, especially its high secondary gain, it was advantageous
to first calculate \( F(n) \), the prepulse height distribution in terms of the number \( n \) of elec-
trons emitted from the first dynode. The procedure consisted of three steps:

(i) The calculation of \( P_2(n, V) \), the distribution of secondary electrons emitted by the
first dynode as a result of ionization by an electron of energy \( eV \).
(ii) The number of events characterized by the parameter \( V \) was then combined with
\( P_2(n, V) \) to give \( F(n) \).
(iii) The effects of the statistics of multiplication at later dynodes in the electron
multiplier were included to yield \( F(h) \), the prepulse height distribution.

The distribution \( P_1(n, V) \) of secondary electrons emitted by a GaP dynode when
bombarded with a primary electron of energy \( eV \) may be calculated in terms of a model
(Coates 1973a), which includes the production of secondary electrons by backscattered
electrons. This model describes the observed behaviour of GaP dynodes reasonably
well in all the respects required. Over the range of primary electron energies considered,
The prepulse height distribution

it was assumed that the true secondary gain, i.e., that of the fraction \((1 - \eta)\) of electrons not backscattered, was proportional to the energy (Simon and Williams 1968, Coates 1970)

\[ \delta = \frac{V}{\varepsilon} \]

where \(\varepsilon\), the mean energy required to produce one secondary electron, is approximately equal to 20 eV. Backscattered electrons are re-emitted with a fraction \(k\) of their initial energy. The model assumes that the energy lost, \(e(1-k)V\), is entirely converted into true secondary electrons, so that, assuming Poisson statistics for each event,

\[ P_1(n, V) = (1 - \eta) p(n, V/\varepsilon) + \int_0^1 E(k) p[n, (1-k)V/\varepsilon] \, dk \]

where \(E(k)\) is the fractional energy distribution of the backscattered electrons, and \(p(n, \delta)\) is a Poisson distribution with mean \(\delta\).

It was assumed that, after ionization at a point at potential \(V\), the ionizing electron continues with little deflection, having lost energy equal to the ionization potential \(I\) of the atom. The ejected electron and the ion possess negligible energy in comparison. The ionizing electron therefore reaches the first dynode with an energy \(e(V_1-I)\), where \(V_1\) is the potential of the first dynode, while the ejected electron arrives with an energy \(e(V_1-V)\). The total distribution \(P_2(n, V)\) of electrons from the first dynode in the prepulse from the ionization event is therefore given by the convolution

\[ P_2(n, V) = \sum_{j=0}^n P_1(j, V_1-I) P_1(n-j, V_1-V). \]

The ionizing and the ejected electrons arrive at the first dynode at slightly different times, unless ionization takes place at a point with \(V=I\), near the cathode, or with \(V=V_1\), at the first dynode. This expression is therefore correct only if the total charge in each pulse is measured. Between the cathode and the first dynode the transit time difference reaches a maximum of about 2 ns in these photomultipliers. This would have only a small effect if the peak current in a 50 \(\Omega\) system were detected since the output pulse width is then about 4–5 ns.

The next step was to calculate the number of afterpulses formed by ionization events in the potential interval between \(V\) and \(V+dV\). Since the results were applied to tubes in which the majority of afterpulses were produced by \(\text{He}^+\) ions, and the window discriminator was set to select these alone, only one ionic species had to be considered. The extension to several species is obvious. The ionization produced by a single electron in a distance \(ds\) measured along its trajectory is

\[ N(V)\,dV = N_\Lambda \sigma_A(V)\, ds \]

where \(N_\Lambda\) is the number density of the atom or molecule ionized, and \(\sigma_A(V)\) the cross section for ionization of \(A\) to the particular species under consideration. For helium, the values given by Rapp and Englander-Golden (1965) were used. This equation may be rewritten as

\[ N(V) = N_\Lambda \sigma_A(V)\, ds/dV. \]

Values for the derivative \(ds/dV\) were obtained from two-dimensional field plots (figure 2), using approximate values for the position and dimensions of the electrode structure ascertained, as far as was possible, from the manufacturer’s literature and by measurement through the tube window. Since the centre of the photocathode was illuminated, \(ds/dV\) was evaluated for electron trajectories from this position only.
A correction was made for the fact that a number of the ions do not produce after pulses large enough to trigger the stop discriminator. If the discriminator is set midway between the peaks in the pulse height distribution produced by single electrons and pairs of electrons emitted from the cathode, an approximate loss function $L(V)$ may be obtained by assuming that ions producing 0 or 1 secondary electrons are not detected:

$$L(V) = 1 - (1 + \delta_A) \exp(-\delta_A)$$

where $\delta_A$ is the secondary gain upon impact of the ion $A^+$ with energy $eV$. Preliminary experiments indicate that $\delta_A$ increased linearly with energy

$$\delta_A = \xi_A V$$

and that for ions incident on bialkali and trialkali cathodes, $\xi_A$ was of the order 0.01 V$^{-1}$. For He$^+$ ions in the 8850 photomultiplier, a value of 0.014 V$^{-1}$ was obtained from the afterpulse height distribution.

The prepulse height distribution $F(n)$ in terms of the number of electrons emitted by the first dynode may therefore be written

$$F(n) = N_A \int_{0}^{T_1} P_e(n, V) L(V) \sigma_A(V) \frac{ds}{dV} dV$$
The prepulse height distribution for one electron leaving the cathode. The upper limit in this integral is open to some doubt. Penetration of the fields from the second and third dynodes into the region in front of the first dynode produces a maximum in $V(s)$ (figure 2). Ions formed after the maximum, $V_{\text{max}}$, where $ds/dV$ is negative, will be accelerated towards the first dynode (Coates 1973b) and will not give afterpulses. However, the ejected electrons formed before the maximum, if $V > V_1$, should not reach the first dynode, although the ion may produce an afterpulse. It was assumed, therefore, that for $V_1 < V < V_{\text{max}}$

$$P_2(n, V) = P_1(n, V_1 - I).$$

Since the calculation was for a single electron from the cathode

$$\sum_{n=1}^{\infty} F(n) = \gamma$$

where $\gamma$ is the ratio of afterpulses to single electron pulses.

To convert $F(n)$ to the observed prepulse height distribution $F(h)$, the single electron response $s(h)$ of the rest of the multiplier, ie excluding the first dynode, was introduced. Then

$$F(h) = \sum_{n=1}^{\infty} F(n) s_n(h)$$

where $s_n(h)$ is the $n$th convolution of $s(h)$ with itself. Since a similar expression could be written for the single electron response

$$S(h) = \sum_{n=1}^{\infty} P_1(n, V_1) s_n(h)$$

the experimental curve for this distribution could be used to give $\eta$ and $\epsilon$, and also some information about the function $E(k)$. Whatever the shape of $s(h)$, for large values of $n$ $s_n(h)$ tends towards a normal distribution. It was sufficient therefore to specify $s(h)$

![Figure 3. Comparison of measured and calculated single electron response.](image-url)
in terms of a scale factor, which enables the experimental curve to be fitted to $S(h)$ along the $h$-axis, and its variance. The latter was calculated from the expression given by Woodward (1948), using values for the dynode gains taken from the measured current gain. The fit between the calculated and measured curves is shown in figure 3. It was assumed for this calculation that $E(k)$ was independent of $k$; although this gave a reasonable fit to the measured curve, it led to a value of $\eta (0.33)$ which was about 20% too high. This result was expected since, in practice, $E(k)$ is found to decrease as $k$ tends to zero (Gomoyunova and Letunov 1965). The first dynode gain was found to be 31, which corresponds to a value for $\epsilon$ close to 20.

4. Discussion and conclusions

The experimental prepulse height distribution for group B afterpulses (figure 4) was found to be very similar to the single electron response. The extra electron from ionization was not detected, although it is possible that a very precise experiment might show a small increase in the peak pulse amplitude. The experimental and theoretical prepulse height distributions for group A afterpulses were in moderate agreement. While both differed from the single electron response in that the probability of observing small pulses was lower, and the peak of the distribution occurred at a distinctly higher amplitude, the theoretical distribution was somewhat broader than that measured. It is probable that this difference was simply the result of a combination of systematic experimental errors and the crudity of some of the assumptions and approximations made in the calculation. For example, it is possible that some group A afterpulses might have been rejected by the window discriminator, so that a section of the corresponding prepulse height distribution is missing. Also, it has been shown that the secondary gain of GaP dynodes is not strictly linear with primary electron energy (Coates 1970), and
this would narrow the theoretical curve for \( F(h) \) slightly. However, the least accurately known term in the final expression involved the variation of potential along the electron trajectory, and hence \( ds/dV \). The procedure adopted to evaluate \( ds/dV \) was clearly inadequate, and no estimate of its error could be made. Fortunately, the shape of \( F(h) \) was not too dependent upon this function. However, many of the other approximations worked well, partly because the functions involved are flat over much of the voltage range, and only vary rapidly close to the cathode or the first dynode, where the probability of afterpulse formation was low.

The shape of \( F(h) \) in this region depends to some extent upon the division of the energy between the ionizing and ejected electrons. If the excess energy of the primary electron, above that required for ionization, were more evenly divided between the two electrons, the calculated distribution would indeed be narrower. Also, any effects from the transit time difference between the two electrons would be reduced. However, the experiments performed by Ehrhardt et al (1971) on the ionization of helium by electrons of 100-600 eV energy indicated that the maximum energy transfer is of the order of a few tens of eV at the most, and this is insufficient to explain the narrowing observed.

The equations do not contain any unknown variable except the helium number density, \( \bar{N}_{\text{He}} \). This may therefore be calculated from the afterpulse ratio, which was equal to \( R(h) \) extrapolated to zero pulse height. The value obtained for \( \bar{N}_{\text{He}} \) for the 8850 photomultiplier used in these experiments was \( 5 \times 10^{13} \text{ cm}^{-3} \), which is close to the natural density of helium in the atmosphere. This figure was regarded as exceptional, however, and due to the operation of this tube in an environment contaminated with helium.

References


Gomoyunova MV and Letunov NA 1965 Sov. Phys.–Solid St. 7 311–5, 316–20


Rapp D and Englander-Golden P 1965 J. chem. Phys. 43 1464–79

Simon RE and Williams BF 1968 Trans. IEEE NS-15 (3) 167–70