Initial state and collective observables in heavy ion collisions

Karolis Tamošiūnas
Universidad tecnica Federico Santa Maria
SILFAE 2009
Final state and collective observables in heavy ion collisions

Karolis Tamošiūnas
Universidad tecnica Federico Santa Maria
SILFAE 2009
Outlook

• Heavy ion collisions at high energies
• Collective observables
• Relativistic Hydrodynamics
• Freeze-Out
• Analytic Landau hydrodynamics and elliptic flow
RHIC experiment

CM energies: 19.6-200 GeV/Nucleon
Species: p+p, d+Au, Cu+Cu, Au+Au
Why heavy ions?

- Phase transition (cross-over?) to unexplored phase of matter with color degree of freedom
- Possibility of QGP in the early universe
- No unique theory to describe QGP, thus theoretical development is necessary.
Collective Observables - 
rapidity distribution

Net-protons rapidity distributions for different beam energies.

Rapidity distribution for different particles in Au-Au collisions, data from BRAHMS.
Collective observables - elliptic flow

\[
\frac{dN}{d\varphi} = \frac{N_0}{2\pi} \left[ 1 + 2v_2 \cos 2(\varphi - \Psi_R) \right]
\]

\[
v_2 = \langle \cos(2(\varphi - \Psi_R)) \rangle
\]

Azimuthal distribution of particles (pions) at 40AGeV NA49

Centrality 10%-30%
Evolution in HIC

Two nuclei collide semi-transparently

Detected particle spectra
Evolution in HIC

Two nuclei collide semi-transparently

System thermalizes

System is too dilute to be in equilibrium

Detected particle spectra

TIME

0.2-1fm/c
Evolution in HIC

Two nuclei collide semi-transparantly

System thermalizes

Initial stage

Fluid dynamical evolution

System is too dilute to be in equilibrium

Freeze-Out

Pressure gradients develop via expansion

Phase transition, hadronization

Resonance decays

Particle emission

Detected particle spectra
Main definitions

For ideal fluids energy-momentum tensor gets following form:

\[ N^\mu(x) = (n(\vec{x}, t), \vec{j}(\vec{x}, t)) = \int \frac{d^3p}{p^0} p^\mu f(x, p) \]

\[ T^{\mu\nu} = \int \frac{d^3p}{p^0} p^\mu p^\nu f(x, p) \]

Boltzmann transport equation, governing change of the distribution function:

\[ p^\mu \partial_\mu f(p) = \frac{1}{2} \int 12 \mathcal{D}_4 f(p_1) f(p_2) W^{pp4}_{p_1p_2} - \frac{1}{2} \int 2 \mathcal{D}_3 f(p) f(p_4) W^{pp4}_{pp2} \]

\[ 12 \mathcal{D}_3 = \frac{d^3p_1}{p_1^0} \frac{d^3p_2}{p_2^0} \frac{d^3p_3}{p_3^0} \]
Relativistic Fluid Dynamics

From kinetic theory. Boltzmann Transport Equation for the evolution of phase-space distribution:

$$p^\mu_k f_k,\mu = \sum_{l=1}^{N} C_{kl}(x, p_k)$$

Then using microscopic conservation laws in the collision integral $C$:

$$T^{\mu\nu},\mu = \sum_{k=1}^{N} T_{k}^{\mu\nu},\mu = 0. \quad N^{\mu},\mu = \sum_{k=1}^{N} N_{k}^{\mu},\mu = 0.$$

These conservation laws are valid for any, eq. or non-eq. distribution, $f(x,p)$. These cannot be solved, more information is needed!

Boltzmann H-theorem: (i) for arbitrary $f$, the entropy increases,
(ii) for stationary, eq. solution the entropy is maximal, $\Rightarrow \exists$ EoS

$$P = P\ (e,n)$$

Solvable for local equilibrium!
Computational Fluid Dynamics

Conservation Laws:

\[ [N^\mu d\sigma_\mu] = 0 \quad \text{and} \quad [T^{\mu\nu} d\sigma_\mu] = 0, \quad [A] = A - A_0 \]

Entropy must not decrease:

\[ [S^\mu d\sigma_\mu] \geq 0 \]

• Viscosity exists in the CFD, because of the mesh - local equilibrium is assumed in every cell.

• Fluid Dynamics does not work for systems away from local equilibrium!

• FD is applicable at the middle stages of a heavy ion reaction.
Freeze Out hypersurface as a discontinuity

Fluid Dynamics

\[ R_1 \]
\[ R_2 \]
\[ R_3 \]
Negative contribution in the Cooper-Frye formula

\[ N = \int_S N^\mu d\sigma_\mu \quad N^\mu = \int \frac{d^3p}{p^0} p^\mu f(x, p; T, n, u^\nu) \]

\[ E \frac{dN}{d^3p} = \int_S f(x, p; T, n, u^\nu) p^\mu d\sigma_\mu \]

\[ p^\mu d\sigma_\mu \] for space-like hypersurface can be positive or negative

**The way out:** modifying the distribution function

1. Cut-Juttner distribution: \[ f(x, p) = f^{\text{Juttner}}(x, p) \Theta(p^\mu d\sigma_\mu) \]

Making The Canceling Juttner distribution

In the Reference Frame of the Front (RFF)

\[ f_{\text{CJ}} = (f_R^{\text{Juttner}} - f_L^{\text{Juttner}}) \Theta(p^\mu d\sigma_\mu) = \]

\[ = \frac{1}{(2\pi\hbar)^3} \left( \exp \frac{\mu - p^\mu u_{\mu}^R}{T} - \exp \frac{\mu - p^\mu u_{\mu}^L}{T} \right) \Theta(p^\mu d\sigma_\mu) \]

where \( u_{\mu}^R = (\gamma, \gamma v, 0, 0) \) and \( u_{\mu}^L = (\gamma, -\gamma v, 0, 0) \).
The CJ distribution with different initial velocities

\[ v_0 = 0.7 \text{ [1/c]}, \Lambda_B = 200 \text{ Me}\zeta, T_0 = 60 \text{ Me}\zeta, v_0 = 1 \text{ [\mu^-]} \]

\[ v_0 = 0.85 \text{ [1/c]}, \Lambda_B = 200 \text{ Me}\zeta, T_0 = 60 \text{ Me}\zeta, v_0 = 1 \text{ [\mu^-]} \]

Distribution curves are in arbitrary units ( \( T=1, m=1 \) )
Gradual Freeze Out in a layer

‘Pre’: All particles interact in the thermal equilibrium. FD is applicable.

‘Post’: Fully frozen out matter. Particles move freely towards detectors.

\[
N^\mu = \int \frac{d^3p}{p^0} \, p^\mu \, f(x, p)
\]

\[
N^\mu(x) = N^\mu_i(x) + N^\mu_f(x)
\]

\[
\partial_\mu N^\mu(x) = 0
\]

\[
\partial_\mu N^\mu_i(x) = -\partial_\mu N^\mu_f(x)
\]

Fig.1 Schematic view of the FO layer, described by two 3-dim hypersurfaces in space-time.

\[
N^\mu_i |_{S_1} = N^\mu
\]

\[
N^\mu_i |_{S_2} = 0
\]
Loss and Gain terms and FO probability

\[ f(x, p) = f^i(x, p) + f^f(x, p) \]

\[ p^\mu \partial_\mu f^f = \frac{1}{2} \int 12 \mathcal{D} f_1 f_2 \mathcal{P} f W_{12}^{p^4} \]

No Loss component in \( f^f \)

\[ p^\mu \partial_\mu f^i = -\frac{1}{2} \int 12 \mathcal{D} f_1 f_2 \mathcal{P} f W_{12}^{p^4} + p^0 \frac{f_{eq}^i - f^i}{\tau_{rel}} \]

**FO of the particles from interacting component**

**Redistributing particles in momentum space**

Collisions can be following:

- \( f^i_1, f^i_2, f^f, f^f_4, f^i_4, f^f_4 \)
Covariant Escape Rate

\[ d\sigma^\mu \partial_\mu f^f(x,p) = f^i(\tilde{x}_1,p) \; P_{esc}^* \]

\[ P_{esc}^* = \frac{1}{\lambda(\tilde{x}_1)} \left( \frac{L}{L - x^\mu d\sigma_\mu} \right)^a \left( \frac{p^\mu d\sigma_\mu}{p^\mu u_\mu} \right)^a \Theta(p^\mu d\sigma_\mu) \]

- \( a \) is influencing the FO profile
- \( a < 1 \) no complete physical FO
- \( a = 1 \) complete FO
- \( a > 1 \) power like complete FO

\( \Theta(p^\mu d\sigma_\mu) \) ensures, that FO particles would not come back to the "int" domain.

For simplicity one can use:

\[ \left( \frac{p^\mu d\sigma_\mu}{p^\mu u_\mu} \right) \approx \cos \Theta_p \]

\( P_{esc}^* \) is Lorentz invariant and works for space-like and for time-like hypersurface!
FO temperature for space-like normal

Temperature evolution of the freezing out matter in the finite layer, for space-like direction. \( \nu_0 \) is velocity of the particle in the FO front.
FO temperature for time-like normal

Temperature evolution of the freezing out matter in the finite layer, for time-like direction. $v_0$ is velocity of the particle in the FO front.
The post FO distribution in the RFF

These gradual post FO distributions strongly resemble CJ distribution, obtained from discontinuous FO.
Landau hydrodynamics


Main approximation is to separate longitudinal and transverse expansions

1st step: longitudinal expansion, along \( z \) axis in 1+1-dim motion:

\[
\begin{align*}
\frac{\partial T^{00}}{\partial t} + \frac{\partial T^{01}}{\partial z} &= 0, \\
\frac{\partial T^{01}}{\partial t} + \frac{\partial T^{11}}{\partial z} &= 0,
\end{align*}
\]

\((t, z, x, y) \equiv (x^0, x^1, x^2, x^3)\)

The \( p \) (or \( e \)) at the center of transverse region is \( e = 3p \) and zero at the surface \( a/2 \)

\[
\begin{align*}
\frac{\partial T^{02}}{\partial t} + \frac{\partial T^{22}}{\partial x} &= 0, \\
\frac{4}{3} \epsilon u^0 u^0 \frac{2x(t)}{t^2} &= \frac{\epsilon}{3a/2}
\end{align*}
\]

The switch from 1st to 2nd stage depends on \( y \)

\[
x(t) = \frac{t^2}{4au^0 u^0} = \frac{t^2}{4a \cosh^2 y}
\]
When \( x(t)=a \) at \( t=t_{FO} \) we switch to new solution: FO, where rapidity is frozen for \( t \leq t_{FO} \).

For central collisions solution gets form:

\[
t_{FO}(y) = 2au^0 = 2a \cosh y
\]

For peripheral collisions:

\[
t_{FO}(y, \phi) = \frac{a(\phi)}{a} 2a \cosh y
\]

Freeze-Out time \( t_{FO}(y) \) depends on rapidity, it is Landau’s FO hypersurface.

Rapidity distribution, dependant on the emission angle:

\[
\frac{dN}{dy} \sim \exp \sqrt{[y_b + \ln(a(\phi)/a)]^2 - y^2}
\]
Elliptic flow for different rapidities

Au-Au at $s_{NN}=200^2$GeV, $y_{beam}=5.29$
The rapidity distribution for different energies is given by:

$$\frac{dN}{dy} \propto \exp\left\{ \sqrt{(y_b + \zeta)^2 - y^2} \right\}$$

$$\zeta = -\ln C_{\text{init}} + \ln C_{\text{FO}}.$$
Conclusions

• HIC collective observables must be understood with respect to the FO HS for both space-like and time-like parts.

• Realistic 3+1 dimensional hydro calculations suggest, that sharp, discontinuous Freeze-Out is a good approximation.

• From analytic Landau’s hydrodynamics it follows, that elliptic flow is Freeze-Out dependent observable.

• Connecting experimental results of dN/dy, v2, dN/dp_t with Landau’s method it should be possible to conclude about influence on the coefficient